derivation for the potential of a point charge above thin layer of uniform dielectric above an anisotropic substrate bulk material

using the furrier transform

apply boundary conditions as and as

applying the conditions ,

and ,

(2)

(1)+(5)

(1)-(5)

(4)

(8)+(3)

(3)-(8)

sub 9 in to 6

sub 10 in to 7

sub 11 in to 12

let h=0

which is the result which is expected when the layer is no longer present

let

result as expected when the layer depth goes to infinity

let

only issue here is that h is 2 times what is expected

let

all trivial cases return the expected results

sub 14 in to

let

test cases

let

returns expected result

let

returns expected result

let

returnes expected result however (z+ would be normal expected result

let

returns expected result

therefore integral is

contour integration and the residue theorem

letting ,

therefore

using result for table of intergrals

trivial checks

let h=0

let

let

result as expected

let

as expected for